Optimization Problems in Production and Planning: Approaches and Limitations in View of Possible Quantum Superiority

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Abstract. Modern production and process planning is characterized by complex and diffuse interrelationships of parameters, properties and control values. New materials, innovative production technologies, differing degrees of automatability and application dependency form a multidimensional problem space for optimization, which cannot be efficiently solved by todays technologies. Approximations in form of genetic algorithms, different heuristics and simplifications exist, but lack applicability due to high runtime and estimation errors.

Quantum computers, quantum annealers and hybrid algorithms show potential to offer added value and better performance over established approaches for optimization, planning and production control, but are often incomprehensible for production engineers. Based on an analysis of industrial problems in different domains and a definition of relevant problem cases, the potential of quantum systems for optimization in production and planning is explored. An approach to close the gap between classical and quantum optimization from an engineering standpoint is made by describing the transformation process for a real-world problem and discussing performance indicators of model implementations.

Keywords: Production planning, Optimization, Quantum algorithms

1 Introduction

Globalized markets with international competition and shorter lifecycles of product variants put pressure on modern production companies to optimize production and process costs. Reactivity in operations and in the planning of processes and production programs is a significant lever for reducing the overall cost. Due to increasing digitization of production, interconnected machines and digital interfaces for all partners in the supply chain, countless pieces of information are available for decision-making. Owing to the sheer volume and the complex inter-relationships, information can ling since not be interpreted and processed by humans. Instead, the modeling of individual problems using formal methods is required to optimize relevant parameters. [1]

Depending on the specific use case, the corresponding number of set parameters, the dynamic input and output variables and the employed modelling formalism, both expressiveness and computation time of the chosen optimization approach vary vastly.

[2] For industry-relevant optimization problems, performance and capacity of common off-the-shelf (mass-market) computing hardware is most often insufficient. Various heuristics and approximation approaches exist on the level of modelling scenarios, and with regards to the mathematical methods employed to perform the actual optimization. Yet, they come with different advantages and drawbacks. [3] During the last years, quantum computers and quantum annealers (despite still being at an early technological stage from an application perspective) have reached sufficient maturity to explore possible approaches to problems inspired from industrial use-cases. In particular, quantum optimization methods can be mapped to multi-order optimization problems in production as well as process planning and control. In this paper, we gently introduce general principles of possible quantum approach, as they differ from known and widely used techniques in many important aspect, and assess their potentials by highlighting advantages, disadvantages and predicted computational capabilities in contrast to classical approaches to industry-relevant optimization problems. In Chapter 2, we describe classical optimization problems in the field of planning and production, and exemplify how to mathematically model such problems. Chapter 3 reviews various established classical approaches to solve optimization problems in production and manufacturing using approximation techniques or heuristics. Chapter 4 then classifies and reviews possible quantum approaches before Chapter 5 shows how to cast an exemplary optimization problem based on a real word application into the mathematical formalism that underlies two seminal approaches to quantum optimization.

2 Standard optimization problems in planning and production

Optimization problems in planning and production can be divided into different problem classes depending on their time horizon and the sector in view. In the area of planning, available resources are distributed according to applicable constraints and demands. Constraints can be process-related (e.g., assembly sequence), time-related (e.g., shelf life), organizational (e.g., personnel availability) or technological (e.g., setup sequences, maintenance downtimes). Optimization problems that have received consideration in the literature stem from a wide range of aspects:

- Resource allocation [4]
- Scheduling [5]
- Capacity planning [5]
- Supply chain optimization [6]
- Layout optimization [7]
- Quality control [8]
- Maintenance planning [9]

In production optimization, more specific variants of the above general problem classes are must be considered; among many other examples, the following topics have been considered in the literature:

- Optimization of cellular manufacturing systems [10]
- Flexible job scheduling problem [11]
- Lot size optimization for production [12]
- Lot size and preventive maintenance [13]
- Robot path optimization [15]
- Energy optimization [14]

Solving all the aforementioned problems with numerical methods requires, of course, to formulate them as mathematical models. Most of the underlying decision problems are NP-complete, as they can be solved in polynomial time (increasing in the size of the input, for instance constrains or variables) on a non-deterministic Turing machine. An elaborate classification of the complexity of the associated optimization variants is known [15], and many of them can be efficiently approximated at the expense of solution quality. Some can be solved by linear programming (resource allocation and quality control) or be formulated as convex optimization problems (robot path and energy optimization) under certain conditions, which allow for efficient numerical solutions or approximation. Often, seemingly small details in formulation and boundary conditions can lead to substantial differences in computational complexity.

An example for an industry relevant NP-hard problem, the capacitated economic lot-sizing with constant setup cost, zero holding cost, nonincreasing production cost and nonincreasing capacities, can be cast in the following form [16]:

$$\vartheta(P) = min \sum_{t=1}^{T} [s_t \delta(x_t) + p_t(X_t) + h_t(I_t)] \tag{1}$$

such that

$$I_{t-1} + X_t - I_t = d_t, \qquad t=1,2,\ldots T$$
$$X_t \leq C_t, \qquad t=1,2,\ldots T$$
$$\delta(X_t) = \begin{cases} 1 \; if \; X_t > 0, \\ 0 \; otherwise \end{cases} \qquad t=1,2,\ldots,T$$
$$I_t, X_t \geq 0, \qquad t=1,2,\ldots,T$$

In the above formulae, X_t, I_t, C_t, d_t and s_t denote production quantity, ending inventory, available capacity, the demand and setup costs for period t.

Exact solutions for NP-complete and NP-hard problems such as (1), as they appear in production planning, cannot be inferred efficiently. Instead, one has to resort to approximate solutions or heuristics.

3 Heuristics for solving optimization problems

Genetic algorithms are well-known approaches [17] modelled on the processes arising in natural selection and genetics. The approach represents a schedule as a series of numbers (chromosomes) that correspond to a sequence of tasks, and uses genetic operations such as selection, cross-over, and mutation to generate new solutions. [18]. In each iteration that applies these operations, the algorithm evaluates the suitability of each schedule by calculating the objective function and selects the best schedules to use for generating the next generation. Over time, the algorithm converges to a near-optimal solution as the best-performing schedules are recombined and improved through genetic operations. The class of genetic algorithms is particularly useful in planning problems where the optimal solution is difficult to find using conventional algorithms, such as problems with many variants and a large number of variables.

Ant Colony Optimization (ACO) is an alternative optimization algorithm based on the foraging behavior of ants. In planning problems, ACO can be used to find the optimal solution for planning tasks by modeling the problem as a search for the shortest path through a graph. The ants, represented as agents in the algorithm, construct solutions by dropping virtual pheromones on the edges of the graph that represent paths between tasks. The algorithm uses this information to update the probabilities with which the ants choose particular paths, leading to exploration of the solution space and convergence toward the optimal solution. The algorithm continues until a satisfactory solution is found or another stopping criterion, such as a maximum runtime, is met [19].

More approaches to solve optimization problems that have been considered in the literature include:

- Particle Swarm Optimization [20]
- Bees Algorithm [21]
- Water-Flow-like-Algorithms [22]
- Multiagent models [23]
- Multivariant Bayesian control methods [24]
- Petri Nets [25]

All of these algorithms trade a decreased possibility of reaching the optimal solution (or deterministically reaching a non-optimal solution) for a runtime improvement over competing deterministic algorithms. However, for large problems with a significant number of variables and parameters, execution on sufficiently powerful and cost-appropriate hardware the runtime may still be in the order of hours or even days, which can be too costly for many scenarios of practical interest.

Recently, quantum algorithms (pure , hybrid or annealing approaches) have been devised [26–28] that are known to exhibit possible computational advantages over classical approaches for certain optimization problems, given widely accepted complexity-theoretic assumptions. While actual quantum advantage on real machines – that suffer from noise and imperfections perturbing computational operations as well as the information represented in quantum bits, feature very limited availability of resources, and are therefore referred to as noisy, intermediate-scale quantum (NISQ) machines – has not been observed so far except for specially crafted, artificial problems [29] they are nonetheless seen as potential future solutions to optimization problems of practical interest [30]. Given that possible advances in hardware or improved algorithmic insights make it hard to predict when actual quantum advantages will be available, it seems reasonable to consider how problems can be cast such that they allow us to process them on future quantum machines, and identify quantum computational primitives that could find deployment in production logistics and manufacturing tasks.

4 Potentials of quantum inspired algorithms in optimization of planning and production

In the past years, many approaches to quantum computing based on entirely different physical principles – from superconducting circuits via trapped ions and neutral atoms to optical techniques – have matured from foundational physical experiments to commercially available systems that are accessible to non-experts via remote cloud access, for instance via commercial offerings by Amazon, IBM, and other vendors. Many quantum algorithms have been devised [31].Unfortunately, while it is possible to prove computational speedups over the best possible (or known) classical approaches for some of them, these advantages do not yet manifest on currently available NISQ systems. Other algorithms are explicitly adapted to the limitations of NISQ systems, and can be executed by them with reasonable quality; unfortunately, it is still unclear if and under what circumstances they can outperform any of their classical alternatives. Figure 1 classifies a list of known algorithms by their problem domain and problem class.

There are many physical implementations of quantum computers that utilize different quantum degrees of freedom to represent and transform information, and even their basic approach to computation differs (applying gates, applying measurements in certain orders, or transforming systems in appropriate ways). Yet, they can exploit non-classical features like entanglement, superposition, and interference that are widely believed to provide improved computational power over classical approaches [32]. Most quantum algorithms that can be applied to scheduling problems operate by encoding the constraints and objectives of the scheduling problem into a specific mathematical objective function, whose variables can be encoded into quantum states. The variables are binary, and formulas can contain up to quadratic contributions in the variables. Additionally, it is not possible to specify explicit constraints that apply to the objective function -- these need to be implicitly included in the formula. The resulting optimization problem (choose values for the variables that minimize or maximize the objective function) is then called a quadratic constrained binary optimization (QUBO) problem -- we provide an example for this class of problems below.

QUBO problems are amenable to solving by quantum annealers, such as those developed by D-Wave Systems [33, 34]. These devices are special-purpose quantum optimizers designed to find an assignment of variables that corresponds to a global minimum of a QUBO formula. Intuitively (and roughly speaking), the method relies on the physical principle that sufficiently slow (“adiabatic”) transformations of physical systems that start in an energetic ground state remain in the ground state; by choosing an initial system whose ground state can be determined and prepared, and by slowly transforming it into a system that represents the problem of interest, a ground-state solution to the latter can be inferred. [35] (note that efficiently finding minimum energy states of physical systems is a classically intractable problem as well). Exact performance guarantees for annealing are not available except in special cases, and determining advantages over classical heuristics is, especially under the influence of noise and imperfections, an open research topic; nonetheless, also quantum annealing is believed to outperform classical optimization approaches in the long run.

Another example of a quantum algorithm that can be used to solve QUBO problems (albeit it also extends to more general tasks) is the quantum approximate optimization algorithm (QAOA). It has been applied to problems such as job shop scheduling, resource-constrained project scheduling, and task scheduling. [36] It is a hybrid quantum-classical algorithm, meaning that it combines quantum computing with classical computing to find an approximate solution to an optimization problem. The algorithm uses a certain problem-specific sequence of parameterized quantum gates to evolve the initial state of a system towards a state that encodes the optimal solution. After sampling the outcome (which is given by a statistical distribution, and makes multiple computational runs and measurements necessary to estimate the probability distribution), a classical optimization procedure is used to update gate parameters such that the next computational run delivers results closer to a desired optimum. This combination of quantum and classical computations is iterated until convergence to a sufficiently good result. While exact performance guarantees for QAOA are not yet known except for very special cases [37], it has been established that classically simulating the algorithm is impossible (again, based on reasonable complexity-theoretic assumptions), and the strong belief prevails that QAOA will lead to performance advantage on sufficiently reliable and scalable quantum hardware. Other quantum algorithms, such as Grover's algorithm, have also been proposed for scheduling problems, albeit these rely on perfect quantum hardware, and only deliver a quadratic speedup for exponential search spaces. [38]

5 Casting a real world problem for quantum algorithms

To illustrate the steps required to cast optimization tasks in form of a QUBO problem, let us commence with a specific example, a job shop scheduling problem that is based on a real-world scenario.

Suppose a company in the toy industry picks and places approximately 1.000 individual single orders per day (J) on three flexible assembly lines (M) in one shift (28.800 s, T). Each order consists of a number (varying between 1-5) of individual articles to be picked (o). Due to the cost structure and lead time promises to customers, the orders should be processed as fast as possible across all three lines (goal function).
Additionally, the following constraints are given:

- Every job must start and run exactly once.
- Only one job can be running on each line at any given time.
- The order of operations within a job is irrelevant to its duration.

To formulate the QUBO, the use case has to be cast in binary variables. For the specific use case, this can be achieved by using the equation detailed in [39]:

$$\min \left(\sum_{j=1}^{J} \sum_{m=1}^{M} \sum_{t=0}^{T} (t + d_{j,m}) \times x_{j,m,t}\right. \tag{2}$$

$$+ p_1 \times \sum_{j=1}^{J} \sum_{m=1}^{M} \left(\sum_{t=0}^{T} x_{j,m,t} - 1\right)^2 \tag{3}$$

$$+ p_2 \times \sum_{j=1}^{J} \sum_{(m_1,m_2)\in P_j} \sum_{t_1+d_{j,m_1}>t_2} x_{j,m_1 t_1} \times x_{j,m_2 t_2} \tag{4}$$

$$+ p_3 \times \sum_{m=1}^{M} \sum_{j_1 \neq j_2} \sum_{t_1<t_2<t_1+d_{j_1,m}} x_{j_1,m,t_1} \times x_{j_2,m,t_2} \tag{5}$$

$$+p_4 \times \sum_{j=1}^{J} \left(\sum_{t=0}^{T} \left(t \times x_{j,\sigma_M^j,t} \right) + d_{j,\sigma_M^j} + \hat{s}_j - \sum_{j=1}^{J} \sum_{m=1}^{M} \sum_{t=0}^{T} (t + d_{j,m}) \times x_{j,m,t} \right)^2) \tag{6}$$

$\hat{s}_j \in \mathbb{Z}, x_{j,m,t} \in \{0,1\}, \forall j = 1,\ldots,J, \forall m = 1,\ldots,M, \forall t = 1,\ldots,T$

Line (2) of the equation describes the completion time of all operations, while the additional parts represent the different constraints (one start (3), precedence (4), no overlap (5) and max make-span (6) constraint) weighted by penalty coefficients (p1 – p4). Note that while manually determining QUBO representations may be a somewhat challenging task, automated means of appropriately transforming various constrained optimization problem formulations that domain experts may be better accustomed to have become available, for instance quark [40]

To solve the formulated optimization problem using quantum algorithms, further transformation steps are required. The QUBO must be mapped to the physical quantum computer or annealer, for which automated approaches are available on quantum annealers [41] and gate-based computers [42]. Typically, this process induces significant overhead in the number of required qubits.

This process of mapping the optimization problem to the physical quantum systems is where the gap between today's optimization in planning and production and quantum potentials persists. For instance, current-generation quantum annealers feature several thousand qubits, yet only scale to problems with 10s or hundreds of variables. It is needless to say that such small-scale problems can at the moment also be solved today using performant hardware and deterministic algorithms in acceptable time.

Hybrid approaches to optimization, like the DWave Leap Hybrid solver, divide problems into sub-parts, and combine part solutions obtained by the quantum. That way, they can scale to substantially larger problem instances, but the additional overhead – both in splitting and recombining the problem, and in the time required for communication. Their approximated solutions are similar in quality to heuristics in standard approaches [43, 44], although a close and nuances look is required to characterize performance and achievable qualities. It has been shown that hybrid solvers can come close to a satisfiable solutions in industry relevant optimization scenarios [45].

Supported by the continuing momentum of developments and progress in the field of hybrid approaches as well as the physical systems, one can assume that the solution of optimization problems on a larger scale may become possible in the future through quantum systems.

6 Conclusion and outlook

Classical, hybrid and quantum approaches each have their strengths and weaknesses when it comes to solving optimization problems in production and planning. Deterministic approaches and heuristics are well-established and often provide exact or nearexact solutions to problems. These methods are often used in production and planning where there is little room for error.

Quantum approaches, such as quantum annealing and QAOA, offer the potential for significant speedups in solving certain optimization problems, however the technology is still in its early stages. Nevertheless, an increasing number of examples, like [46], show the applicability of quantum technologies to real problems in production and planning.

In order to solve more problems with large number of variables and complexity efficiently, quickly and optimally in the future, we expect the use of quantum technologies to become a viable option. With the increasing establishment of physical systems, growing performance and hybrid approaches, the interfaces and application possibilities of the technologies will also increase. Even if practical application have not yet materialized, we belief that it is worth while to start investigating the use of novel computational approaches to problems in our domain.

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