


Quantum Dark Magic

Efficiency of Intermediate Non-Stabiliserness

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I. POSTER TITLE

Quantum Dark Magic: Efficiency of Intermediate Non-Stabiliserness

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III. POSTER ABSTRACT

Abstract—While quantum systems are known to possess inherent computational advantages over classical computers, constructing algorithms that harness such advantage remains an open challenge. Non-stabiliserness (i.e., traversal of states outside the Clifford orbit), is a necessary condition, as de-quantisation is otherwise possible. Nevertheless, an excess of non-stabiliserness is also known to not be advantageous.

In this paper, we present an approach to understanding the efficient use of non-stabiliser states by tracking their behaviour across various algorithms. Our techniques reveal different efficiencies in the use of non-stabiliserness, leading us to hypothesise that greater classical optimisation degrees of freedom can introduce unnecessary non-stabiliser consumption, which becomes costly with error correction.


IV. POSTER RELEVANCE

Index Terms—quantum resource theory, quantum magic states, state evolution, intermittent entanglement, quantum algorithms

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I. INTRODUCTION

Quantum computing (QC) extends the classical computational model, rather than replacing it. Quantum computations contain classical parts [7, 2], which renders it challenging to distinguish the respective contributions to computational power. Identifying inherently quantum parts in computations is crucial to understanding the chances and limitations of quantum approaches. In this paper, we address this question using geometrical distance arguments within a solution space. Entanglement is a fundamental computational resource of QC [4], but its effect on computational power is not easy to characterise. We adopt Stabiliser-Rényi-Entropies (SRE) as a measure of non-stabiliserness, or “magic”, in quantum states [6]. Using SRE, we locate where non-classical effects appear during the execution of contemporary quantum algorithms and quantify the efficiency of non-stabiliser consumption using a geometric distance measure.

II. NON-STABILISERNESS

Before defining a measure for non-stabiliserness, it seems pertinent to clarify the concept itself. Let \mathcal{C}_n be the Clifford group, which can be efficiently simulated classically. Then STAB is given by the orbit of \mathcal{C}_n . In [6] the Stabiliser-Rényi-Entropy $\text{SRE}_\alpha(|\psi\rangle)$ was defined, which measures the non-stabiliserness of $|\psi\rangle$.

The main characteristic of Stabiliser-Rényi-Entropies is that a state $|\psi\rangle$ is in STAB if and only if $\text{SRE}_\alpha(|\psi\rangle) = 0$. It can

be shown that Stabiliser-Rényi-Entropies are invariant under Clifford operations, that is, if $U \in \mathcal{C}_n$ then $\text{SRE}_\alpha(U|\psi\rangle) = \text{SRE}_\alpha(|\psi\rangle)$ for an arbitrary state $|\psi\rangle$.

III. GEOMETRIC PERSPECTIVE

Given a random Haar sampled quantum state $|\psi\rangle$, then $\mathbb{E}_{|\psi\rangle \sim \text{Haar}}(\text{SRE}_\alpha(|\psi\rangle)) \in \mathcal{O}(n)$ [3] and $\text{SRE}_\alpha(|\psi\rangle) \leq \log(2^n) \in \mathcal{O}(n)$ [6]. This implies that, although intermediate states with $\text{SRE} \geq 0$ are both indicative of and indeed necessary for quantum advantage, their occurrence is not exceptional. This raises the question whether the observed non-stabiliserness contributes to the computation, or if it is merely a by-product of a suboptimal choice of unitary propagators.

What is the meaning of *contributing to the computation*? Geometrically, quantum state evolution is equivalent to rotating a state vector. While a circuit represents a singular unitary and thus a direct rotation to its final state, its concrete realisation most likely diverges from this direct path at some point. In geometric terms, the shortest, most direct state evolution path is characterised by the geodesic from the initial state to the target. In [1], Anandan and Aharonov presented exactly this geometric perspective in conjunction with the concept of *geodesic efficiency* $\mu_{\text{gd}} = s_0/s$ of a state evolution where s_0 is the geodesic distance and s the actual distance travelled.

a) *Problem Hamiltonian*: Usually, there is more than one unique solution to a computation problem; all possible solutions comprise the *target space*. Instead of rotating the initial state to one specific target state $|\psi_{\text{T}}\rangle$, a quantum algorithm can reach any state within the *target space*. Thus, the geodesic distance s_0 needs to be reinterpreted as shortest geodesic distance to one of the target space states. Let \mathcal{T} be the subspace containing all superpositions of valid problem solutions and let H_c be a Hamiltonian defined as a projector onto \mathcal{T} . Then

Theorem 1. *Given a target space \mathcal{T} and the corresponding problem Hamiltonian H_c projecting onto \mathcal{T} , we have*

$$s_0(\mathcal{T}) := \min_{|t\rangle \in \mathcal{T}} s_0(|t\rangle) = 2 \arccos\langle H_c \rangle \quad (1)$$

can be shown (see the preprint Ref. [8] for a proof).

b) *Permutations*: In a typical quantum circuit, qubits are sequentially numbered, which is a naming convention without physical significance. Qubits q_i and q_j could be remapped $q_{\sigma(i)}$ and $q_{\sigma(j)}$ for some permutation $\sigma \in S_n$. The ordering, which can be performed by an efficient Clifford circuit, does not alter non-stabiliserness. Therefore, it can be ignored in the analysis of non-stabiliserness resource consumption. Given an operator $\hat{\sigma}$ that permutes the qubits according to $\sigma \in S_n$, then $\text{SRE}_\alpha(\hat{\sigma}|\psi\rangle) = \text{SRE}_\alpha(|\psi\rangle)$ [8]. To factor out such inconsequential permutations, we define an equivalence relation $|\psi_l\rangle \sim |\psi_r\rangle$ that is satisfied iff there exists a $\hat{\sigma}$ such that $\hat{\sigma}|\psi_l\rangle = |\psi_r\rangle$. Let $[[\psi]]$ be its according equivalence class, with $[\mathcal{T}] = \bigcup_{|t\rangle \in \mathcal{T}} [[t]]$. Then it immediately follows that $\text{SRE}_\alpha(|\psi\rangle) = \text{SRE}_\alpha([[\psi]])$. For the geodesic distance, we need to extend the definition to equivalence classes to: $s_0([[\phi]]) = \min_{|\phi'\rangle \in [[\phi]]} s_0(|\phi'\rangle)$ and $s_0([\mathcal{T}]) = \min_{|t\rangle \in \mathcal{T}} s_0([t])$.

IV. EXPERIMENTS

State evolution techniques can be broadly grouped into two categories: *structured* and *unstructured* state evolution. The former introducing few restrictions on the circuit logic, and leave more freedom to the optimisation step. The latter directly impose the problem structure onto the circuit and significantly reduce the number of free parameters. For each ansatz, we solved 20 SAT instances with the circuits spanning $n = 7$ qubits and $p = 7$ layers. Every instance was randomly sampled with a clause variable ration of $|C|/|V| = 3$, which generates SAT instances that are constrained enough to be at the start of the easy-to-hard phase transition.

We can now show how our method reveals actual differences in structured and unstructured state evolution techniques. We chose the seminal NP-complete problem of Boolean satisfiability (SAT) based on 3-CNF formulae $F : \mathbb{F}_2^n \rightarrow \mathbb{F}_2$ (see Ref. [5] for more details).

a) *Structured and Unstructured State Evolution*: In an unstructured state evolution ansatz, a generic circuit template leaves maximal flexibility to be adjusted later in a classical optimisation step that minimises a cost function. An optimal value is achieved if the final state is in the target space \mathcal{T} . As an exemplary ansatz, we chose a hardware-efficient VQE architecture.

In contrast to unstructured state evolution techniques, a structured ansatz already is infused with instance information. One can show that problem structures extrapolated from common instance structures are sufficient to successfully approximate the state evolution of such methods [5]. As a representative for structured state evolution, we chose a standard QAOA approach where the driving problem Hamiltonian is the problem Hamiltonian H_c defined above.

b) *Results and Comparison*: Comparing structured and unstructured state evolution, we notice that the former more directly approaches the target space, while the latter behaves more erratic. To analyse individual computational steps, we calculate the delta of s_0 before and after each step. We observe

a positive correlation between distance reduction and non-stabiliser consumption *only* for the structured evolution. This indicates that, speaking on a per-step basis, the structured ansatz more efficiently approaches the target.

V. CONCLUSION

In this paper, we extended geodesic distance measures to evolutions targeting a complex target space, deriving the distance from a Hamiltonian's expected value. We applied this framework to analyse structured and unstructured state evolution, finding that the structured approach is more geodesic-efficient. By combining this with Stabiliser-Rényi-Entropy, we showed that the structured ansatz is significantly more efficient in consuming non-stabiliser resources. Our approach demonstrates the potential of combining resource theoretic and geometric methodologies. For a complete discussion we refer to our preprint in Ref. [8].

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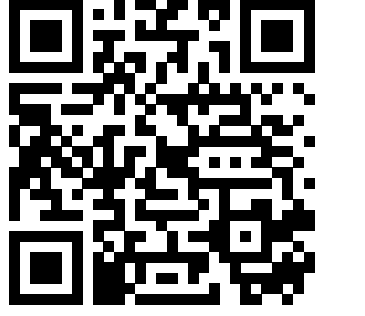
Quantum Dark Magic Efficiency of Intermediate Non-Stabiliserness

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- Quantum computations contain classical parts
- Where in the quantum circuit do actual quantum computations happen?

- Non-Stabiliserness serves as a measure for quantumness
- Geometric perspective allows for quantification of non-stabiliser efficiency



Preprint [1]

Non-Stabiliserness

Definition: [2]

$$\text{SRE}_\alpha(|\psi\rangle) = \frac{1}{1-\alpha} \log \sum_{P \in \mathcal{P}_n / \{\pm \mathbb{1}_n\}} \Xi_P^\alpha(|\psi\rangle) - \log 2^n \quad (1)$$

$$\Xi_P(|\psi\rangle) = \frac{1}{2^n} \langle \psi | P | \psi \rangle^2 \quad (2)$$

Theorem: [2] A state $|\psi\rangle$ is in STAB if and only if $\text{SRE}_\alpha(|\psi\rangle) = 0$.

- SRE invariant under Clifford operations
- $\text{SRE}(|\psi\rangle) > 0$ for all $|\psi\rangle$ not in STAB

High non-stabiliserness is common [3]

Geometric Perspective

State evolution traces a path over surface projective Hilbert space $\mathcal{P}(\mathcal{H})$ (manifold)

Shortest path from $|\psi_0\rangle$ to $|\psi_T\rangle \rightarrow$ geodesic distance s_0

Efficient state evolution follows geodesic path [4]

Theorem: Given a target space \mathcal{T} and a Hamiltonian H_c projecting onto it, then

$$s_0(\mathcal{T}) := \min_{|t\rangle \in \mathcal{T}} s_0(|t\rangle) = 2 \arccos \langle H_c \rangle$$

Permutations

Qubits can be arbitrarily reordered with only Clifford operations

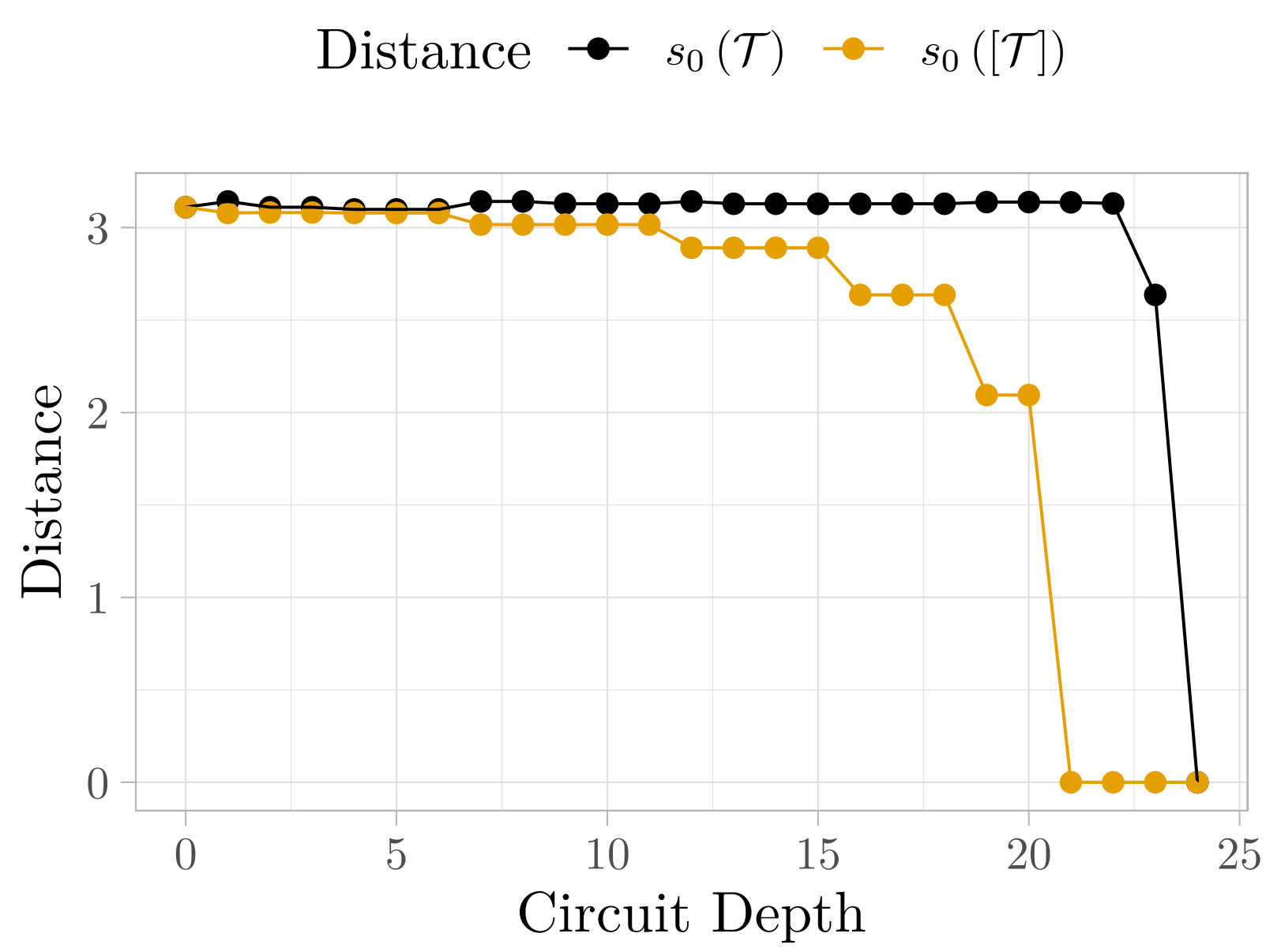
\rightarrow SRE consumption invariant under qubit order

Definition: Let \mathcal{S}_n be the set of all qubit permutation operators on n qubits, then

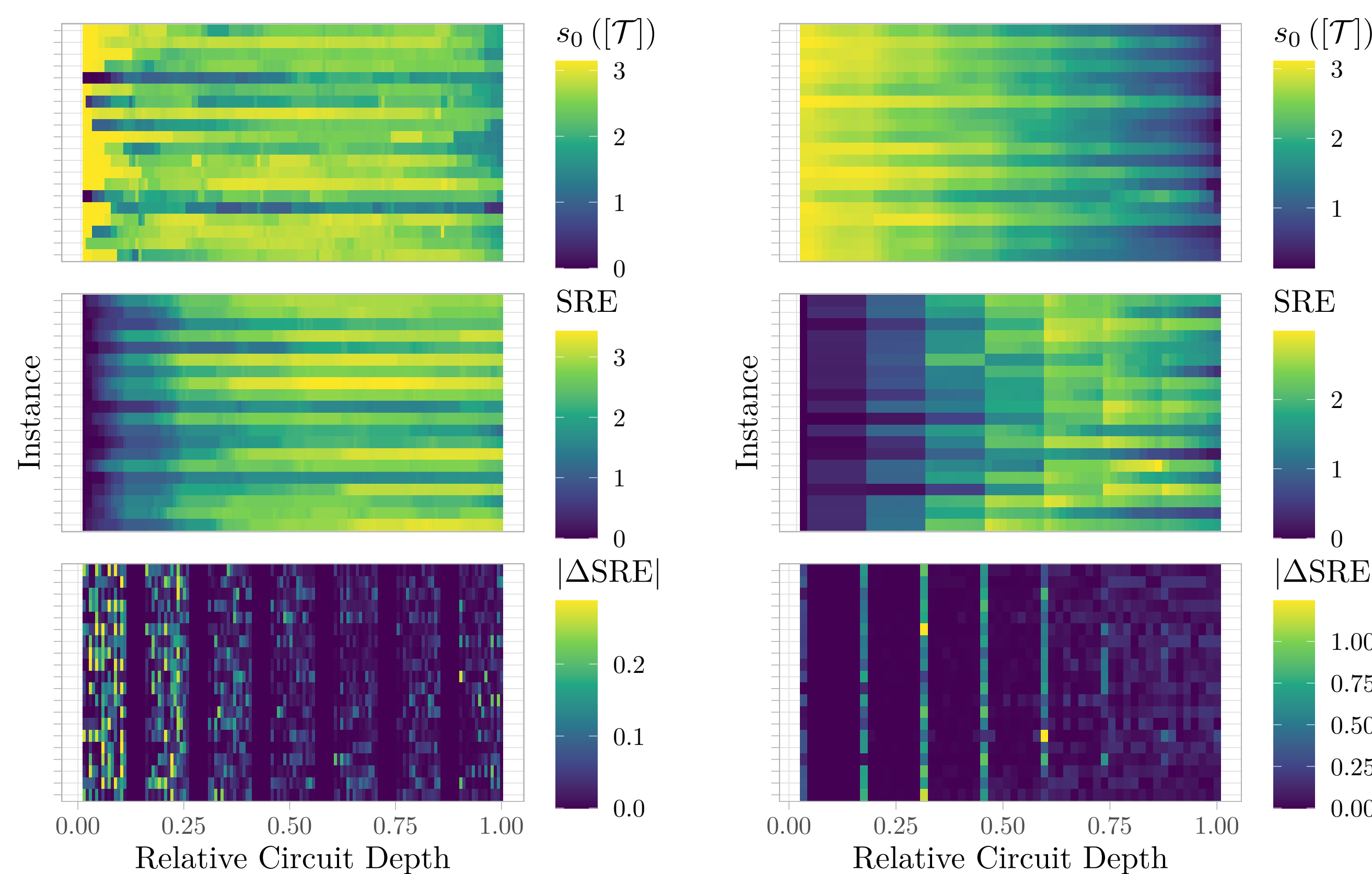
$$[|\psi\rangle] = \{\hat{\sigma}|\psi\rangle : \forall \hat{\sigma} \in \mathcal{S}_n\} \quad \text{and} \quad [\mathcal{T}] = \bigcup_{|t\rangle \in \mathcal{T}} [|t\rangle]$$

further

$$s_0([|\phi\rangle]) = \min_{|\phi'\rangle \in [|\phi\rangle]} s_0(|\phi'\rangle) \quad \text{and} \quad s_0([\mathcal{T}]) = \min_{|t\rangle \in \mathcal{T}} s_0(|t\rangle)$$

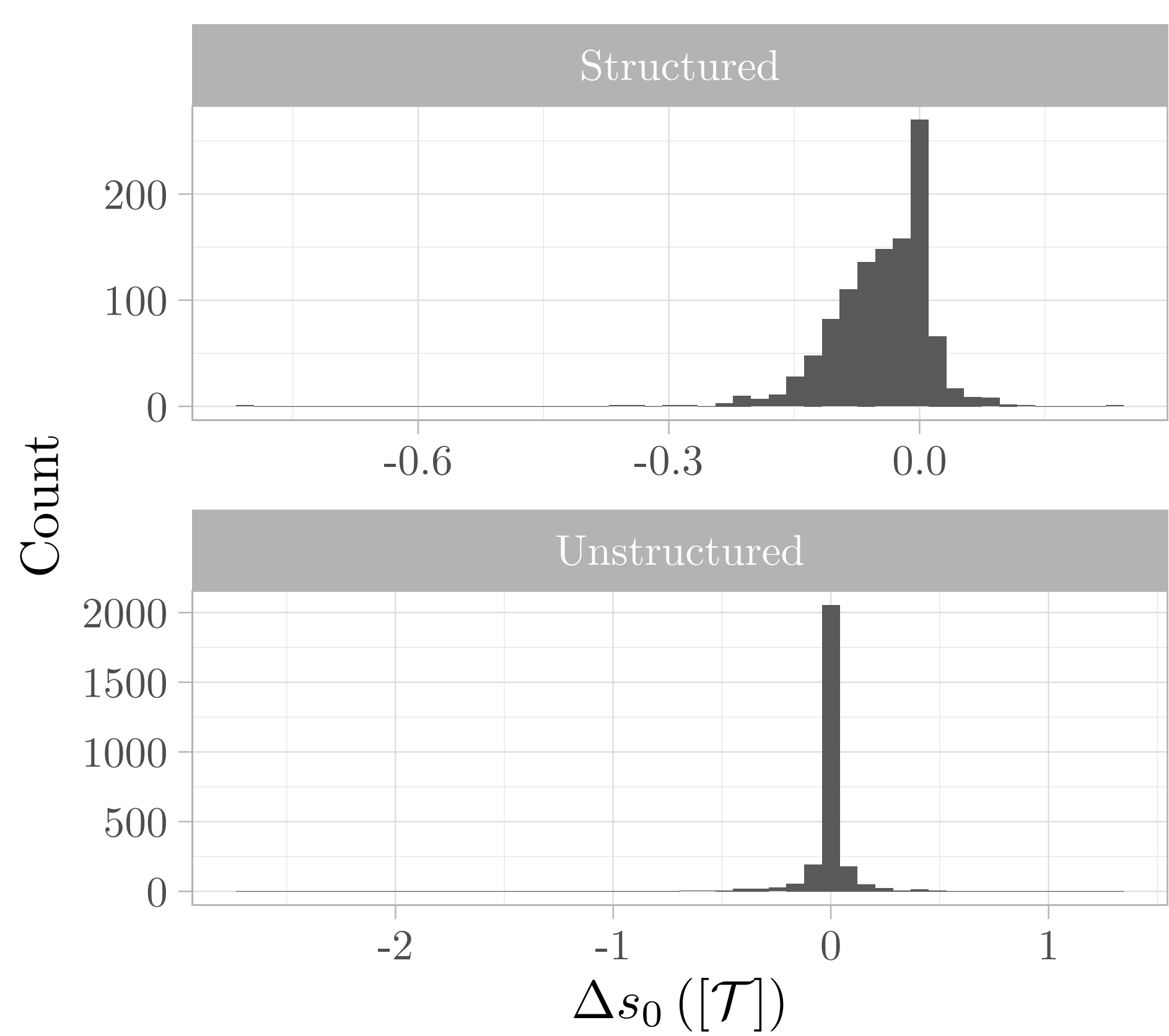


For a QFT circuit, calculating the minimal geodesic distance over all target space permutations $s_0([\mathcal{T}])$ reveals non-Clifford computational progress being made before the final qubit order reversal. In the direct distance to the target space $s_0(\mathcal{T})$, without taking permutations into account, these effects are not visible. This demonstrates how potential non-stabiliser effects can be masked by measures that are not Clifford agnostic.

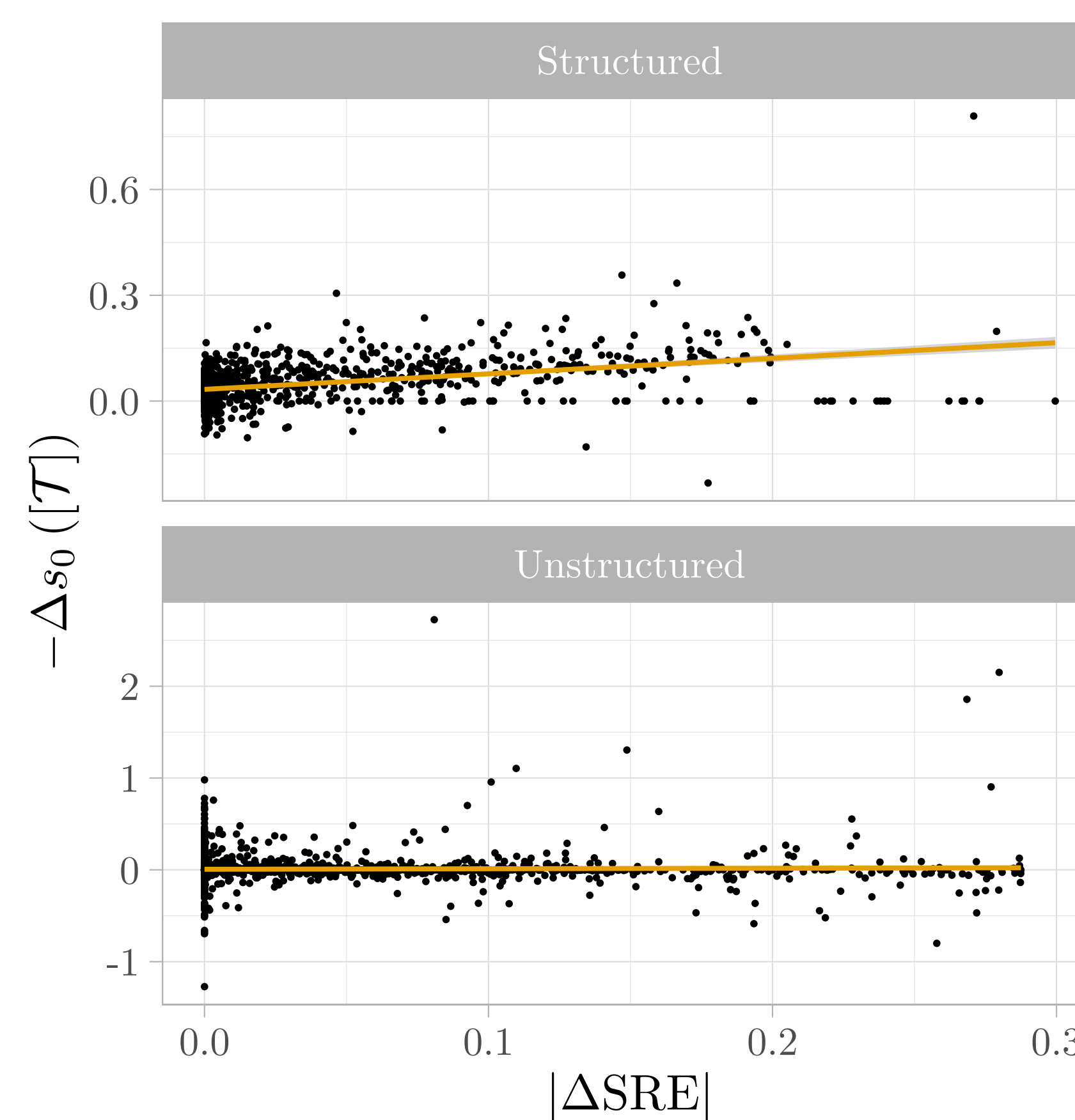


(a) All three plots show different aspects of intermediate states (x-axis) in a state evolution using an **unstructured** ansatz solving instances (y-axis). Under this ansatz the state approaches the target space quite erratically apparent by the uneven jumps in the geodesic distance from $[\mathcal{T}]$ depicted in the **top** figure. Even though the non-stabiliserness (**middle**) seems to grow more smoothly, we can see that its consumption expressed as the delta (**bottom**) also takes place in a quite chaotic pattern.

(b) All three plots show different aspects of intermediate states (x-axis) in a structured state evolution using a **structured** ansatz solving instances (y-axis). We can see that under the QAOA ansatz the state evolution smoothly approaches the target space $[\mathcal{T}]$ (**top**). The non-stabiliserness build up (**middle**) is highly structured peaking at ≈ 0.75 relative circuit depth, after which it starts to smoothly decrease again. This is also reflected non-stabiliser consumption patterns shown in the **bottom** plot.



The distribution of in- and decreases of the distance to the target space ($x \sim \pm \Delta s_0([\mathcal{T}])$) is heavily skewed towards the decreases for the structured ansatz (top), with the majority of distance changes being negative. In comparison, ignoring few outliers below 0, the distance change distribution is centred around 0 for the unstructured ansatz (bottom).



Structured state evolution (top) shows clear correlation between non-stabiliser consumption $|\Delta \text{SRE}|$ and steps reducing the geodesic distance to the target space $-\Delta s_0([\mathcal{T}])$. In contrast, we cannot observe a similar correlation for the unstructured ansatz (bottom). Data are restricted to $|\Delta \text{SRE}| < 0.3$ to filter outliers (this retains approximately 98% of the original data points).

Conclusion

Geometric distance

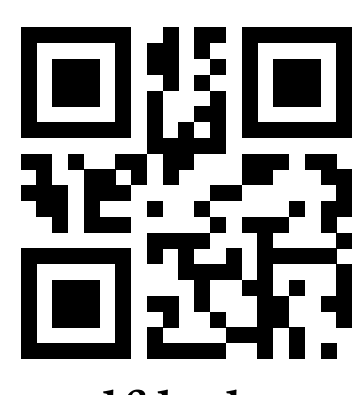
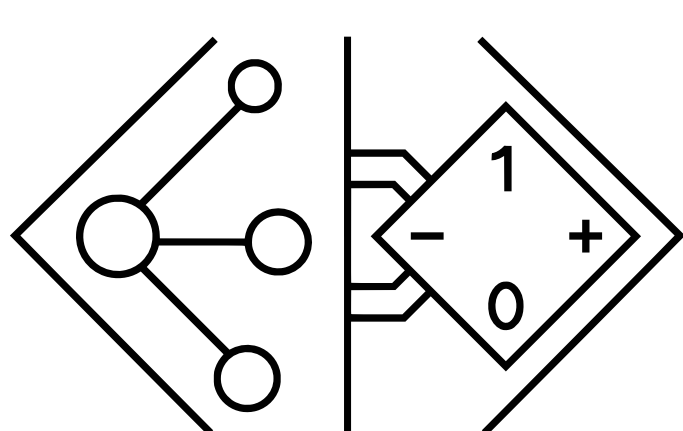
- Extension of geodesic distance measure from distance to state to more complex target space \mathcal{T}
- Geodesic distance to \mathcal{T} can be derived from expected value of problem Hamiltonian
- Qubit order agnostic geodesic distance measures

Quantum Fourier transform:

- Actual quantum computational progress shadowed by final swap gates
- Permutation agnostic distance measure reveals shadowed computational progress

Structured vs unstructured state evolution:

- Combination of resource theoretic SRE measure and geometric distance measures
- Qualifying efficiency of resource consumption
- Structured ansatz significantly more efficient than unstructured ansatz



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